Big Bang Cosmology, Part 2: Nucleosynthesis and the Background Radiation

ELEGANT CONNECTIONS IN PHYSICS

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This installment of *Elegant Connections* continues the story of big bang cosmology. The first article described the expansion of the universe.[1] Here I'll describe how the first nuclei were formed. When the temperatures were so incredibly high in the very early times, no nuclei could form at first, despite the abundance of neutron-proton collisions, because they would be immediately photo-fissioned in the bath of energetic photons. The story of primordial nucleosynthesis and the story of the photon gas are intimately connected.

One important connection between photons and other particles arises through pair production and annihilation. When the photons (denoted g) have sufficient energy, their collisions can produce a particle P and its antiparticle P'. Likewise, a particle-antiparticle pair can collide and annihilate into a pair of photons. Hence the very early universe witnesses a dynamic equilibrium between the photons one one hand, and particle-antiparticle pairs on the other, through the reaction

$$P + P' \leftrightarrow g + g \,. \qquad (1)$$

As the universe expands and cools, the production of particles of mass m is quenched when the thermal energy $k_B T$ (where k_B denotes Boltzmann's constant) drops below the particle's mass energy mc^2. This defines a "threshold temperature" T_m for that particle,

$$T_m = mc^2/k_B \,. \qquad (2)$$

The mass of a proton or neutron (collectively called "baryons") is about 1GeV/c^2, corresponding to a baryon threshold temperature of 10^{13}K. Let's also recall a result of Part 1, where we derived the temperature T as a function of time t:[1]

$$T^2 t \approx 1.9\times10^{20}\ K^2 s \,. \qquad (3)$$

Eq. (2) predicts that the temperature of the universe dropped to the baryon threshold temperature about $t = 10^{-6}$s after the big bang. After this time, pair annihilation continued, but without pair creation to replenish the supply of baryon-antibaryon pairs. Why then didn't all the baryons annihilate into a gas of photons? The fact that the universe today consists of *matter*, with very little *antimatter*, suggests a lack of symmetry between the reactions of matter and antimatter!

This story, which occured at something like the first millionth of a second, forms another tale of its own. For now, let us be content with a simple illustration: suppose that, left to itself, particle P is stable, but that its anti-particle P' can decay, with a tiny probability ë, into other particles: $P' \to a+b$. The probability of P' *not* decaying is 1−ë. Every PP' annihilation eliminates one P and one P' and produces two photons, but every P' that first decays into some other product, $P' \to a+b$, no longer remains available to annihilate with any P particle. As a result, after all the P' particles have annihilated there will be some P particles left over. Such reactions *are indeed observed*, for example in the decays of the unstable strongly interacting particles called "kaons." Their decays set a precedent for breaking the symmetry between particles and antiparticles, suggesting the reality of this mechanism for producing a universe of matter, devoid of antimatter.

We shall see that primordial nucleosynthesis calculations fit observation well, provided that for every billion antiprotons there were about a billion-and-one protons:

$$\begin{aligned}(\#\text{protons}) / (\#\text{antiprotons}) \\ &= (1 + 10^9)/10^9 \\ &= 1 + 10^{-9}\end{aligned}$$

and similarly for the neutrons and antinuetrons. Then every billion antiprotons annihilating a billion protons leaves behind a solitary proton, creating at least two billion new photons and raising the a photon-to-baryon ratio h from unity to about 10^9.

Let us begin our investigation of nucleosynthesis from about t = 0.01s, when these baryon-antibaryon annihilations are finished. From now on the early universe consists of a relativistic gas inhabited by the photons, the surviving protons and neutrons, plus leptons such as the electron, its neutrino, and their anti-particles.[2]

At t = 0.01s, the surviving protons and neutrons are

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about the same in number. They can change back and forth into one another via the two-body collisions with leptons,

$$p + e^{-} \leftrightarrow n + \text{þ} \quad (4)$$

and

$$p + \text{þ'} \leftrightarrow n + e^{+} . \quad (5)$$

These reactions form examples of the “weak interaction,” a family of reactions that includes beta decay. Because the neutron’s mass is about 1.3MeV/c^2 greater than the proton’s, the neutron also beta-decays into a proton (plus an electron and an antineutrino):

$$n \rightarrow p + e^{-} + \text{þ'} . \quad (6)$$

We’ll neglect the reverse reaction of Eq. (6), because three-body collisions are rare compared to two-body ones.

Let N_n denote the number of neutrons and N_p the number of protons, and let

$$N_n + N_p = N . \quad (7)$$

In terms of the fractions $X_i = N_i/N$, Eq. (7) says

$$X_n + X_p = 1 . \quad (8)$$

Let’s assume the ultra-relativistic gas of photons, leptons, and baryons in the early universe to be in thermal equilibrium. This means that the probability of a particle having energy E is proportional to the Boltzmann factor, $\exp(-E/k_BT)$. All the particles are free, with energy $E = K + mc^2$, where K denotes the kinetic energy, which has the same average value for particles when the temperature T is uniform. Thus

$$X_n/X_p = exp(-1.3\text{MeV}/k_BT) \quad (9)$$

Eqs. (8) and (9) may be united into

$$X_n = [1+ exp(-1.3\text{MeV}/k_BT)]^{-1} \quad (10)$$

from which we see that X_n is determined by T. Eq. (10) holds so long as free neutrons and free protons form the only possible states of nuclear matter. Note that in the early portion of the epoch under consideration, when T is so large that $1/T$ is indistinguishable from zero, then $X_n \approx X_p \approx ½$.

The collision-induced reactions of Eqs. (4) and (5) cease when the expansion of the universe had thinned the gas to the point that the leptons can no longer find the baryons. Of course, this diminishment of these collisions forms a statistical process, as do all processes we consider. But for the purpose of obtaining simple estimates of what occured, we can think of the cessation of reactions (4) and (5) as a sudden event. Its time and temperature occurs when the lepton-baryon collision rate drops below the expansion rate. The expansion rate is measured by Hubble’s parameter H, and the reaction rate follows from the details of the weak interaction. Let us calculate these rates.

To calculate Hubble’s parameter, from Ref. 1 we recall one of the equations that describes the expansion of the universe (with zero cosmological constant):

$$(dR/dt)^2 + k = (8\mathbf{p}G/3c^2)\mathbf{r}R^2 \quad (11)$$

where $H = (dR/dt)/R$, with R the cosmic scale factor, G = Newton’s gravitational constant, $\mathbf{r}$ = the energy density of the universe, and k = the curvature parameter. At very early times, the particles all move at essentially the speed of light, so that the energy density is given by an extention of Stefan’s law, $\mathbf{r} = g\mathbf{s}T^4/c$ where $g\mathbf{s} \approx 3.4\times10^{-7}$ W/m^{2}K^4 when one takes into account only the electron, proton, neutron, neutrino, photon, and their antiparticles (g counts all spin degrees of freedom). In the very early universe, k is negligible even if k Þ 0, and we find

$$H^2 = (8\mathbf{p}G/3c^3)g\mathbf{s}T^4 .$$

To estimate the rate $\mathbf{G}$ of reactions (4) and (5), imagine a baryon to be a target of cross-sectional area A. This is not necessarily the same area as the baryon’s geometric silhouette, because the interaction between the baryons and leptons has an energy-dependent range. Our baryon target moves at essentially the speed of light, so in time dt it sweeps out a cylinder of volume $Acdt$. The number of reactions it suffers with the leptons per unit time will therefore be the product of Ac and the number of leptons per unit volume n, or

$$\mathbf{G} = nAc . \quad (12)$$

From statistical mechanics, we learn from the Fermi-Dirac distribution the number density of ultra-relativistic leptons to be about $(k_BT/\backslash c)^3$. From the “weak interaction” that describes Reactions (4) and (5), one learns that the cross-section $A \approx (G_F\, k_BT\backslash c)^2$ where G_F = Fermi’s constant = 10^{-5} GeV^{-2}. From Eq. (12) we now have

$$\mathbf{G} = (k_BT/\backslash c)^3(G_F\, k_BT\backslash c)^2c$$
$$= G_F^2\,(k_BT)^5 / \backslash . \quad (13)$$

When we equate the expansion rate to the reaction rate, $\mathbf{G} = H$, Eqs. (11) and (13) yield $T \approx 1\times10^{10}$K, which according to Eq. (2) occurs at t = 1.9s. By Eq. (10), we find that X_n = 0.18 at this time.

After 1.9s, the number of neutrons decreases, and the number of protons increases, thanks to the $n \rightarrow p$ decay of

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Eq. (6). The half-life of the neutron is about 887s. Thus at times t > 1.9s, but before nucleosynthesis, the fraction of baryons that remain as neutrons declines according to

$$X_n = 0.18\ exp[-(t-1.9s)/887s]. \qquad (14)$$

The fraction of baryons that are protons grows as $X_p = 1 - X_n$. This situation continues until protons and neutrons fuse to form stable nuclei. To that problem we turn next.

Fusion to Helium

To build up the elements beyond hydrogen-1 requires the fusion of protons and neutrons. Before that can occur, the temperature must drop to a value where the photons, which outnumber the baryons by a billion to one, no longer have sufficient energy to photo-fission any newly formed nuclei.

Nucleosynthesis begins when, for example, a proton and a neutron fuse to form a nucleus of hydrogen-2, the deuteron:

$$n + p \rightarrow {}_1H^2 + g\ . \qquad (15)$$

To build helium, the deuteron absorbs another proton to form a nucleus of helium-3 and a photon. These reactions must happen twice before the next reaction can happen once: when two helium-3 nuclei collide, two protons are thrown off, leaving a nucleus of helium-4. So the overall reaction that begins with *pn* fusion and ends with a helium-4 nucleus is

$$2n + 2p \rightarrow {}_2He^4 + 4\ g\ . \qquad (16)$$

The weak link in this chain of reactions exists in hydrogen-2, the deuteron. This nucleus provides the intermediate state towards building heavier nuclei (even when two protons fuse, one proton immediately turns into a neutron by emitting a positron and neutrino). But the deuteron is a fragile nucleus, with a binding energy of only 2.2 MeV. So with one neutron per deuteron, to avoid photofissioning the new deuterons, nuclear fusion must wait until *the number of photons with energy greater than 2.2 MeV drops to less than the number of neutrons*. Let v denote the fraction of photons having energies ≥ 2.2 MeV. These photons are found out in the high-energy tail of the Planck distribution (see "blackbody" radiation in the textbooks). The number of photons with energies greater than 2.2 MeV will be vN_g, where N_g denotes the number of photons, and thus our condition for the onset of nucleosynthesis becomes

$$vN_g = N_n\ , \qquad (17)$$

i.e., the fraction of photons with energy sufficient to photo-fission the deuteron equals the neutron-to-photon ratio. Now the photon-to-baryon ratio is $h = N_g / (N_n+N_p) = 10^9$, and the neutron-to-baryon ratio is $X_n = N_n/(N_n+N_p)$. Thus Eq. (17) becomes

$$v = X_n / h = 10^{-9} X_n\ . \qquad (18)$$

Both v and X_n are functions of temperature and thus the time, with X_n given by Eq. (14). An evaluation of v follows from the Planck distribution for the thermodynamics of a gas of photons. This distribution describes the energy density dr_g/dw of the photon gas in the frequency interval [w,w+dw]:

$$dr_g/dw = \backslash w^5/c^3(2p)^3[exp(\backslash w/k_B T)-1]^{-1}\ . \qquad (19)$$

A sketch of this distribution function is shown in Fig. 1. It's the area under the curve for which $\backslash w \geq 2.2$ MeV that we're after.

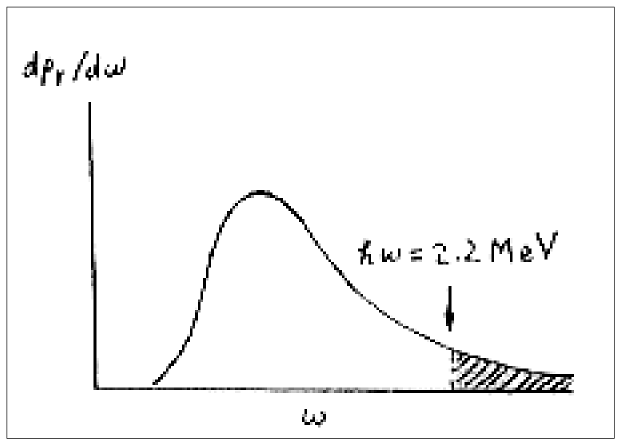

Fig. 1: The Planck distribution, showing the 2.2MeV tail of interest.

To cut a long story short, we find that the temperature where all this happens is about 0.8×10^9K, which by Eq. (2) corresponds to the time t = 297s. Then from Eq. (14) we find that at this time, the fraction of baryons that "freeze out" as neutrons is

$$X_n(297s) = 0.18\ exp[-(297-1.9s)/887] = 0.13 \qquad (20)$$

leaving 0.87 as the fraction of baryons that are protons.

From this moment onward, the proton-to-neutron ratio stands at 0.87/0.13 = 6.7, which means that there are about 134 protons for every 20 neutrons. Almost all of these neutrons will be bound into nuclei of helium-4, since no nuclei with five baryons are stable (the delay in leaping over five-nucleon nuclei allows the universe to expand and thin before much fusion can occur beyond helium). For simplic-

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ity, let us assume that *every* neutron ends up in a helium-4 nucleus.

OK, let's see: for every 134 protons there are 20 neutrons, for a sum of 154 baryons. These 20 neutrons plus 20 protons will make five nuclei of helium-4, consuming 40 of our 154 baryons.[3] Thus the fraction by mass of nuclear matter that is found in the form of helium-4 a few minutes after the big bang is 40/154 = 26%, leaving 114/154 = 74% of the nuclear mass as hydrogen-1 nuclei.

To sum up so far: our simple calculations predict that of the nuclear matter emerging from the primordial fireball, about 26% by mass formed into helium-4, and about 74% as hydrogen-1. When astronomers survey the universe, taking into account the relatively small amount of element-building that has occured after stars began to shine and supernova, they find the universe's primordial chemical composition by mass to be about three-quarters hydrogen and one-quarter helium.[4] More realistic calculations[5] agree even closer with the empirical data; our simple model serves as an upper limit on the helium abundance.

It is interesting to note that the problem was originally turned around: given an estimate of the observed abundances of the elements, George Gamow and others realized about 1948 that the very early universe must have been hot enough for photo-fission to prevent most baryons from being cooked into heavy nuclei.[6] They used the abundances of the elements to predict the background radiation, the photon gas "afterglow" of the big bang itself.

The Background Radiation

Using arguments similar to those where we found the threshold temperature for deutron survival, one can also calculate the temperature below which electrons can remain in stable orbit (with binding energy 13.6 eV) around hydrogen nuclei without being instantly photoionized. This temperature turns out to be about 4000K, which happens about 350,000 years after the big bang. From that moment onward, the universe becomes transparent to photons, and this photon gas continues to cool as the universe expands. Today, some 10^{10} years later, this photon gas should have a temperature of just a few degrees above absolute zero, and be uniformly distributed throughout the universe. *If the big bang really happened, then such a "background" radiation must exist.* In 1965, two researchers working for the phone company with a microwave communications antenna, Arno Penzias and Robert Wilson, discovered this uniform background radiation.[7] Subsequently explored at other wavelengths, this radiation fits precisely a Planck distribution at a temperature of 2.735 K.

This radiation's temperature is uniform across the sky to four decimal places, as one would expect if it all came out of the same "oven" originally. However, small spatial fluctuations in the radiation's temperature should exist for the following reason. In the early universe, matter and radiation were in thermal equilibrium. But today, matter is not uniformly distributed; on the scale of hundreds of millions of light years it appears as superclusters of galaxies. Hence the initial statistical fluctuations in density, which were amplified by the expansion into these clusters, should be imprinted on the radiation. The 1992 measurement to six decimal places by the COBE satellite confirmed these expected fluctuations, $\Delta T/T = 10^{-6}$, a crucial vindication of the big bang model.[8]

Including Other Elements

Our derivation of the primordial element abundances is clearly over-simplified.[9] It is therefore important to appreciate what has been left out, which more realistic models must take into account.

In general, one must include *all* species of nuclei, from bare protons and neutrons, through the isotopes of hydrogen and helium, lithium and beryllium, to isotopes of carbon and beyond. These nuclei turn into one another through various collisions and radioactive decays. Consider nuclear species i, where $N_i/N = X_i$. Let us write a differential equation for the evolution of X_i. Species i will be made by the reactions of other species j, so the increase in X_i will be proportional to X_j. At the same time, species i will diminish in proportion to X_i, because when species i exists it can suffer collisions and decays that turn it into something else. Hence we may write

$$d X_i/dt = \sum_j \lambda(j \rightarrow i) X_j - X_i \sum_j \lambda(i \rightarrow j) \quad (21)$$

where the $\lambda(i \rightarrow j)$ are various reaction rate coefficients that have to be calculated and/or measured from elementary particle physics, nuclear physics, and statistical mechanics. An Eq. (21) exists for *each* nuclear species; hence a very large array of coupled differential equations, with time-dependent rate coefficients, must be solved: a considerable challenge!

We have seen how, in our elementary calculation, we obtain a fairly robust prediction of the hydrogen-1 and helium-4 abundances. Therefore, the real mettle of the realistic calcuations comes in predicting the abundances of the other isotopes, which occur in minute trace amounts. I hope that our oversimplified model provides you with an appreciation of the skills of those who do the serious nucleosynthesis calculations!

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NOTES

[1] Elegant Connections in Physics, Part 1: The Expansion of the Universe, *Radiations*, Fall 2000. Eq. (2) and all other calculations shown in this *Connections* series on cosmology are faithful to the assumptions of our oversimplified pedagogical model (see Ref. 9).

[2] We also ignore the heavier leptons (muon, tau), and the thermodynamic effects of neutrinos. These effects must be included in any more realistic model.

[3] Here we neglect the difference between the neutron and proton masses, and the difference between four proton masses and the helium-4 mass. These differences amount to only a few parts out of a thousand.

[4] K.A. Olive and G. Steigman, "On the Abundance of Primordial Helium," *Astrophys. J*. Supp. 97, 49-85 (1995).

[5] T.P. Walker, G. Steigman, D.N. Schramm, K.A. Olive, and H. Kang, "Primordial Nucleosynthesis Redux," *Astrophys. J*. **376**, 51-69 (1991); K.A. Olive, D.N. Schramm, G. Steigman, and T.P. Walker, "Big-Bang Nucleosynthesis Revisited," *Phys. Lett*. B236, 454-460 (1990). See also S. Weinberg, *Gravitation and Cosmology*, Ch. 15, Wiley (1972); John A. Peacock, *Cosmological Physics*, Cambridge (2000); B.W. Carroll, D.A. Ostlie, *Modern Astrophysics*, Addison-Wesley (1996). For a popular treatment see S. Weinberg, *The First Three Minutes*, Basic Books (1988).

[6] G. Gamow, "Expanding Universe and the Origin of the Elements," *Phys. Rev*. **70**, 572-573 (1946); R.A. Alpher, H. Bethe, and G. Gamow, "The Origin of Chemical Elements," *Phys. Rev.* **73**, 803-804 (1948).

[7] A.A. Penzias and R.W. Wilson, *Ap. J.* 142, 419 (1965).

[8] "COBE Measures Anisotropy in Cosmic Microwave Background Radiation," *Physics Today*, p. 17-19 (June 1992).

[9] Our treatment here is similar to B. Eskridge and D. Neuenschwander, "A Pedagogical Model of Primordial Helium Synthesis," *Am. J. Phys*. 64, 1517-1524 (1996).

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