Light and Two Curves, One Century Apart

ELEGANT CONNECTIONS IN PHYSICS

by Dwight E. Neuenschwander

In 1900 Max Planck derived from first principles the spectrum of light in thermal equilibrium at temperature T. Planck's curve set the tone for 20th century physics. For example, with his curve the laser was anticipated in 1916; and since the mid-1960's the Planck distribution has revealed the temperature of the cosmic background radiation (CBR).

One century later saw the assemblage of an amazing body of data into a curve that may be remembered as "the cosmic Rosetta Stone."[1] It opens the door to precision cosmology. This curve describes a harmonic series of microKelvin temperature fluctuations in the CBR, fluctuations correlated to galaxy formation. In this issue of *Radiations* that celebrates contemporary applications of light, we review the elegant connection between these "two curves of two centuries." The 1900 Planck curve, and its 2000 descendant that measures fluctuations about it, provide two windows into structure formation in the early universe, at the Kelvin and nanoKelvin temperature scales respectively.

THE THERMODYNAMICS OF LIGHT

In 1900 Planck may have been inspired by concurrent developments in physics and chemistry: the concept of atoms was making its case, matter was *quantized*.[2] It occured to Planck that if radiant energy of frequency $\boldsymbol{w}$ was quantized as well, with $\boldsymbol{w}$ the fundamental mode in a discrete harmonic series, then the troublesome high-frequency overtones would be damped out by the Boltzmann factor $e^{-E/kT}$, yielding finite total energy. So Planck postulated that a harmonic wave of frequency $\boldsymbol{w}$ would have one of the quantized energy levels E_n, where

$$E_n = n \backslash \boldsymbol{w}, \quad n = 0,1,2,3,... \qquad (1)$$

When these states were inserted into the machinery of statistical mechanics, one turned the crank and derived the energy density $\boldsymbol{r}$ of the modes having frequencies in the interval $[\boldsymbol{w}_1, \boldsymbol{w}_2]$ at temperature T:

$$\boldsymbol{r}(\boldsymbol{w}_1, \boldsymbol{w}_2) =$$

$$[(kT)^4/\boldsymbol{p}^2(\backslash c)^3] * _{\boldsymbol{w}_1}^{\boldsymbol{w}_2} ds\, s^3/(e^s - 1) \qquad (2)$$

where s ° $\backslash \boldsymbol{w}/kT$. When summed over all frequencies from zero to infinity, Eq. (2) becomes Stefan's law,

$$\boldsymbol{r} = \boldsymbol{s}T^4 \qquad (3)$$

with $\boldsymbol{s}$ ° $\boldsymbol{p}^2k^4/15(\backslash c)^3 = 5.67\times10^{-8}\ WK^4/m^2$. The Planck distribution function, or energy density per frequency interval, is

$$[d\boldsymbol{r}/d\boldsymbol{w}]_{\text{Planck}} = (\backslash/\boldsymbol{p}^2c^3)\boldsymbol{w}^3[\exp(\backslash\boldsymbol{w}/kT)-1]^{-1} \qquad (4)$$

(see Fig. 1). Planck found that his distribution would fit the empirical spectrum of light provided that \ assumed the numerical value of 1.1×10^{-34} J-s. Planck's 1900 curve formed a turning point. The flowering of 20th century physics begins here, starting with the fundamental concepts of quantum mechanics, to applications that range from lasers to astrophysics. For example, the relation between temperature and wavelength of the brightest color emitted is described by $T\,\boldsymbol{l}_{\text{peak}} = 3\times10^{-3}$ Km, a fact used to measure the surface temperatures of stars.

COSMIC BACKGROUND RADIATION

Early students of big bang cosmology realized that if something like the expanding primordial fireball *really happened*, then a cosmic background radiation *must* exist. Such radiation *does* exist: using a microwave horn antenna that was built to track the Echo satellite, Arno Penzias and Robert Wilson published in 1965 "A Measurement of Excess Antenna Temperature at 4080 MHz," reporting a CBR temperature of 3.5K±1K.[4] The universe evidently began about 15 billion years ago as a relativistic gas of matter and radiation in thermal equilibrium, and has been expanding ever since according to Hubble's law,

$$dR(t)/dt = H(t)R(t), \qquad (5)$$

where t denotes the time since the big bang, as measured by any co-moving observer, H is the Hubble parameter, whose present value is $H_{\text{now}} = 70 \pm 7$ (km/s)/Mpc, and R denotes the "cosmic scale factor" whose ratio at two times describes the factor by which space has expanded. As space *stretches*, so does the wavelength of light; if upon emission the wavelength at the source is $\boldsymbol{l}_{\text{emit}}$, an observer distant from

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the emission event later detects a wavelength that has been stretched into a larger value $\lambda_{obs} \geq \lambda_{emit}$. This expansion is discussed in terms of the "redshift parameter z," defined operationally from

$$1 + z \equiv \lambda_{obs}/\lambda_{emit} \, . \qquad (6)$$

where also

$$R_{now}/R_{then} = 1 + z \, . \qquad (7)$$

Because the early universe cools according to $R \sim 1/T$,

$$T_{now} = T_{then}/(1 + z) \, . \qquad (8)$$

At early times ($t \leq 10^4$ yrs.) the time is approximately related to the temperature by[5]

$$T^2 t \approx 2\times10^{20} \text{ K}^2\text{s} \, . \qquad (9)$$

The neon coffee shop "Espresso" sign above the writer's head has redshift zero; but sources whose light was emitted from the very early universe have $z >> 1$.

The evolution of $R(t)$ is given by Einstein's field equations as applied to a homogeneous and isotropic universe. In "natural units," these equations say

$$(dR/dt)^2 + k = (8\pi G/3)\rho R^2 + \Lambda R^2 \qquad (10)$$

and

$$d^2R/dt^2 = -(4\pi G/3)(\rho + 3P)R \qquad (11)$$

where G denotes Newton's gravitation constant, ρ the energy density, P the pressure, Λ the "cosmological constant," and $k = 0$ or ±1, depending on whether space is flat or curved respectively. The equations of state relating energy density to pressure can be summarized by

$$P/\rho = w \, , \qquad (12)$$

where w is a known model-dependent quantity; for instance, in a gas of hot photons, $w \approx 3$.

The evolution of the CBR splits asunder into times "before" and times "after" an event called "decoupling" (or "recombination"). In the "before" phase, the universe consists of a nucleus-electron plasma in thermal equilibrium with a photon gas. The nuclei and the electrons are strongly coupled to the photons by Thompson scattering, whose relentless photo-ionization prevents the formation of neutral atoms. This turmoil continues until the temperature drops low enough—about 3000K, at $t \approx 300{,}000$ yrs.—when the number of photons with energies greater than the electron binding energy in hydrogen, becomes less than the number of electrons. This takes so much time because the photons outnumber baryons two billion to one. At the moment of decoupling, when atoms form, matter becomes electrically neutral, and the universe abruptly becomes transparent to the photons. The photons then stream freely across the univese, continuing to cool as the universe expands. Since the last-scatter surface at decoupling is now redshifted to $z \approx 1000$, that leftover cosmic glow should have a temperature today on the order of 3K.

When it was first detected, the CBR temperature was *expected* to be *approximately* uniform. The isotropy of the Hubble expansion suggests that each part of the early universe would have been at about the same temperature as every other, and thanks to "inflation" at about 10^{-35}s, the primordial gas would expand rapidly after thermal equilibrium was established. But the existence of non-homogeneous structures such as galaxy clusters also requires that one should look with finer precision for small but significant departures from a *uniform* temperature. Such precision required serious background problems to be overcome. Of them Steven Weinberg wrote in 1972, "These uncertainties will probably be with us until far-infrared measurements can be made with cryogenic equipment carried by artificial satellites."[6]

In 1990 a collaboration led by George Smoot analyzed data from cryogenic equipment carried aboard the COBE satellite. Other than the expected local galactic background and the dipole shift due to our motion relative to the CBR, the COBE group reported a cosmic radiation temperature of 2.728K ± 0.002K. This measurement showed one of the finest agreements between theory and observation in the history of physics (Fig. 1). The theoretical curve was the 1900 Planck curve.[7,8] The COBE instruments found a uniform temperature across the sky to one part in 10^4.

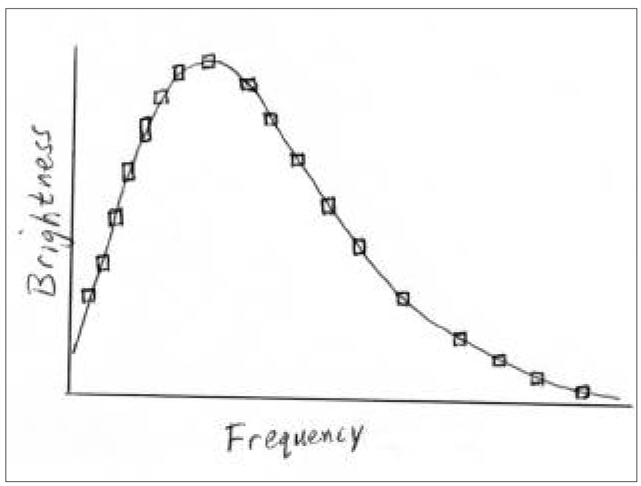

Fig. 1. Sketch of the CBR spectrum, fit to a temperature of 2.73±0.6K.

If the imprint of matter's structure was going to be observed in the radiation, more precision was needed. At higher precision there *had* to be fluctuations in the cosmic radiation temperature, by a straightforward logic: First, the existence of galaxy clusters suggests the existence of primordial density fluctuations. Because of gravity, a local density maxima would accrete even more matter, and local

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density minima would grow less dense as their matter slides into potential wells somewhere else. Second, the fluctuations would be incredibly amplified when the universe inflated in size by many orders of magnitude, some 10^{-35}s after the big bang, and grow into the large-scale structures we see today. Third, such inhomogenities must show up in the CBR, as follows.

Consider a region before decoupling that boasts larger-than-average energy density, whose gravitational field attracts even more matter (especially if the higher densities are seeded by non-baryonic, electrically neutral "cold dark matter," not subject to photo-dissociation). The photons are coupled electromagnetically to the nuclei and electrons; therefore, as the charged matter accretes the photons are squeezed—*their* density and pressure and temperature increase—like a spring, the photons oppose the compression. Given adequate time, the compressed photon gas will make the infalling matter rebound, thereby cooling the mix in the resulting rarefication. Such oscillations—sound waves—will continue until decoupling occurs. These sound waves in the plasma-photon gas travel with velocity v_s, where from the wave equation for a fluid,

$$v_s^2 = \partial \rho / \partial P \ . \qquad (13)$$

In the early universe, $w = 3$ in Eq. (12), which yields, in conventional units,

$$v_s = c/\sqrt{3} \qquad (14)$$

where c is the speed of light in vacuum. Here then lies a testable inference—and a significant signal-to-noise challenge! Experience with waves in other settings suggests that the noise can be resolved into a sum of *discrete* harmonics *if* a fundamental wavelength exists.

Expanding Railway Station

Let me trot out an analogy that I find helpful for visualization. In some science museums you find a set of pipes, each tuned to resonate at a specific frequency. When you take these pipes to a noisy place such as Grand Central Station, and put your ear up to one of them, say the one tuned for 440Hz, you hear that tone coming through beautifully because *all the harmonics are present incoherently*; this pipe merely selects just one.

Furthermore, if sound waves heat the air adiabatically, and we have an infrared camera with sufficient sensitivity and shutter speed, then at a given instant a snapshot of the air would reveal waves of temperature fluctuations about the mean value, $T(x) = T_o + \Delta T(x)$. *The building itself provides a fundamental wavelength*. With opposite walls separated by the length L, the temperature fluctuations form a harmonic series built on the fundamental wavelength $\lambda_1 = 2L$:

$$\Delta T(x) = \sum_{n=1}^{\infty} A_n \sin(nk_1 x) \ , \qquad (15)$$

where $k_1 = 2\pi/\lambda_1$ denotes the fundamental wavenumber. The power carried by the field is proportional to the wave function squared, so to compute the average power $\langle P \rangle$ that fills the station we would calculate

$$\langle P \rangle \sim \int_0^L [\Delta T(x)]^2 \, dx \qquad (16)$$

The orthogonality of the sine functions turns Eq. (16) into

$$\langle P \rangle \sim \sum_{n=1}^{\infty} |A_n|^2 \qquad (17)$$

A power spectrum might look something like Fig. 2.

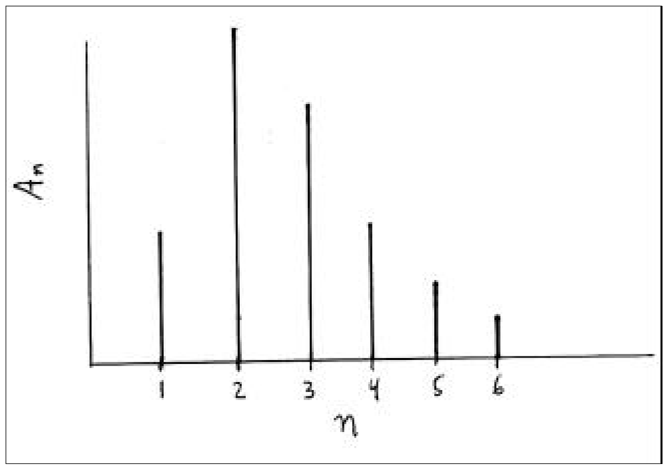

Fig. 2. Example of amplitude spectrum for acoustic waves in a confined space.

Imagine next that this noisy Grand Central Station terminal is *expanding* such that the opposite walls move apart with constant speed v. Let us further imagine the existence of a sudden "decoupling time" t_D, such that when $t \geq t_D$, the noise no longer changes the temperature field. The length L_D of the room at $t = t_D$ will "freeze in" a fundamental wavelength λ_1. This fundamental wavelength also corresponds to the time $t_D = L_D/v = ½ \ \lambda_1/v$. For times $t > t_D$, pictures of the temperature fluctuations will show what they looked like at the decoupling time, but stretched (red-shifted) as the building expands and the temperature uniformly cools.

CBR Harmonic Series

In the early universe, what determines the fundamental wavelength for the CBR temperature fluctuations? The speed of compressional waves in the pre-decoupled medi-

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um is $c/ = 3$. Thus the decoupling time, t_D ' 300,000yr, sets the length scale: $L_D = v_S\, t_D$ ' 2×10^5 light years is the distance between "the walls" *at the time of decoupling*. Today, with that region now at redshift at $z = 1000$, it will have expanded to a size of 2×10^8 light years. The photons from that region have taken the age of the observable universe, about 1.4×10^{10} years, to reach us today. Thus this expanded region, originally of scale L_D, will now appear to us as subtending an angle on the sky of $\boldsymbol{Dq}$ ' 2×10^8 cyr/1.4×10^{10} cyr ' 0.014 rad ' 18. *Therefore*, the sound waves that show up today as temperature fluctuations should exhibit their fundamental mode as a signal at the one-degree angular resolution scale.

What should we expect to see at the 18 scale? The sites where the temperature reaches a maximum will be those where the infalling matter has just reached maximum photon compression at the moment of decoupling. Such sites will have never experienced a rarefication; so this maximum in the temperature fluctuations should occur at the center of a region of our fundamental distance scale L_D. Behold a prediction: the red-shifted CBR today should show temperature fluctuation maxima at the 18 angular size.

Furthermore, if a harmonic structure exists, the overtones should show up at ½8, 1/38, ¼8, ... Physically, the signal at ½8 corresponds to a relative temperature *minimum* (and a second *peak* in the power spectrum, because the power goes as the *square* of the signal), because as a region of size scale ½L_D, upon decoupling the plasma would have experienced one compression and bounced back to maximum rarefication. Similarly, the third harmonic would correspond to a region of size 1/3L_D, in which there was one squeeze followed by one rarefication and another squeeze to a secondary maximum temperature. Hence the odd-numbered peaks will correspond to temperature maxima, and the even-numbered peaks to temperature minima in the fluctuations. Thus will order emerge out of chaos, in the form of peaks in the power spectrum *vs*. angular resolution.

Now we can quantitatively modify our Grand Central Station analogy. We must replace the station x-axis with the two-dimensional celestial sphere, on which we map the temperature field in latitude and longitude. These nanoKelvin fluctuations will be about the COBE value,

$$T(\boldsymbol{q},\boldsymbol{f}) = 2.728\text{K} \pm \boldsymbol{D}T(\boldsymbol{q},\boldsymbol{f}) \,. \qquad (18)$$

The basis functions for a spherical surface are not sines, but the spherical harmonics $Y_\ell^m(\boldsymbol{q},\boldsymbol{f})$. Instead of Eq. (19) we have

$$\boldsymbol{D}T(\boldsymbol{q},\boldsymbol{f}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} A_{\ell m}\, Y_\ell^m(\boldsymbol{q},\boldsymbol{f}) \,. \qquad (19)$$

The average power of the temperature fluctuations are calculated by averaging over the sky,

$$\langle P\rangle \sim * \,[\boldsymbol{D}T(\boldsymbol{q},\boldsymbol{f})]^2\, d\boldsymbol{W} \qquad (20)$$

The orthogonality of the spherical harmonics turns Eq. (20) into

$$\langle P\rangle \sim \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} |A_{\ell m}|^2 \,. \qquad (21)$$

When one also averages over m, one obtains a power spectrum to be fit to data,

$$\langle P\rangle \sim \sum_{\ell=0}^{\infty} |A_\ell|^2 \,. \qquad (22)$$

To relate the values of ℓ to the angular resolution to which they correspond, consider the angles $\boldsymbol{Dq}$ that are subtended between the zeroes of, for instance, $Y_\ell^0(\boldsymbol{q},\boldsymbol{f})$. These angular separations occur at, roughly, 1808/ℓ. So the angular resolution to which the $|A_\ell|^2$ corresponds describes fluctuations that subtend an angle on the sky of about 1808/ℓ. The centers of the peaks expected at 18, ½8, 1/38, ¼8, ... should therefore appear in the multipole expansion near ℓ = 200, 360, 540, 720,...

Starting about 1992 and continuing to this present moment, several groups of observers have impressively measured $\boldsymbol{D}T(\boldsymbol{q},\boldsymbol{f})$ to microKelvin precision. The big breakthrough was in 1992, when George Smoot's group established the *existence* of temperature fluctuations at the tens of microKelvin level.[9] They recorded an rms temperature fluctuation of 30$\boldsymbol{m}$K down to an angular resolution of 78,

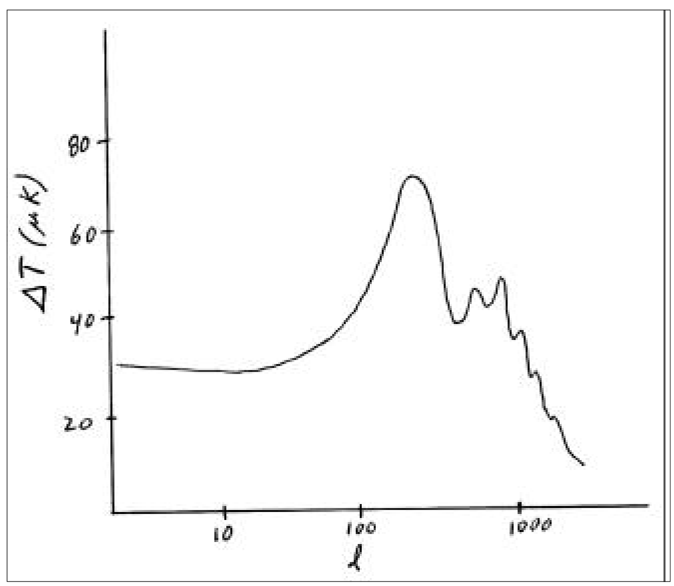

Fig. 3: Sketch of temperature fluctuations in tens of microKelvins, as a function of angular resolution $\boldsymbol{Dq}$ or multipole moment ℓ, where $\boldsymbol{Dq}$ = 1008/ℓ.

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which corresponds to multipoles from $\ell = 0$ to about $\ell = 15$.

More recently, observations made from the balloon experiments of Maxima,[10] Boomerang,[11] and the South Pole interferometer DASI[12], among others, have confirmed the first and second peaks in the CBR temperature fluctuation spectrum, and shown evidence for the third peak. A sketch of these "echoes of the big bang" are shown in Fig. 3.[13]

Centered near $\ell = 200$, or at a resolution of about 18, the expected first peak shows a ***DT*** reaching about 70 ***m***K, and forms the dominant feature of the spectrum.

The placement, widths, and relative heights of these peaks of the CBR harmonic series carry loads of information about the composition, flatness, and age of the universe.[1] For example, in its first season of operation the DASI collaboration recently reported a measurement of ***W***, the ratio of actual to critical energy density of the universe. If ***W*** < 1 (> 1) then the universe will be open (closed); if ***W*** = 1 precisely then the universe is "flat." The DASI investigators report:

The Degree Angular Scale Interferometer (DASI) has measured the power spectrum of the Cosmic Microwave Background anisotropy over the range of spherical harmonic multipoles 100 < ℓ < 900. We compare this data, in combination with the COBE-DMR results, to a seven dimensional grid of adiabatic CBM [cold dark matter] models....We find that the total density of the Universe, ***W*** *= 1.04 ± 0.06, in accordance with the predictions of inflationary theory. In addition we find that the physical density of baryons* [i.e., the contributions of baryons to ***W***] *to be 0.022 ± 0.04, and the* [contribution to ***W***] *of cold dark matter to be 0.014 ± 0.04. The value of* [the baryon's contribution to ***W***] *is consistent with that derived from measurements of the primordial abundance ratios of the light elements combined with big bang nucleosynthesis theory. Using the result of the Hubble Space Telescope Key Project...we find that* ***W****(total) = 1.00 ± 0.04, the matter density* ***W****(matter) = 0.40 ± 0.15, and the vacuum energy density* [due in effect to a cosmological constant, which would make the expansion accelerate with positive sign] *to be* W_L *= 0.60 ± 0.15 (all 68% confidence limits).*[14]

That 60% of the energy density in the universe is evidently some sort of "dark energy" making the expansion *accelerate* has, perhaps, "loosed something brand new and menacing into the world of physics." That it should come through Planck's curve seems in character with that important part of physics, if not in character with Planck's personal tastes. One of his biographers writes:[15]

In 1906 or 1908 Planck had come to see that his compromise over cavity radiation had loosed something brand new and menacing into the world of physics...In 1910 he expressed himself in the manner of a protector of a menaced and even losing cause: "The introduction of the quantum of action h into the theory should be done as conservatively as possible..."

One wonders what Planck would say today about the cosmic background radiation and "dark energy." Occasionally introducing "something new and menacing into physics" keeps physics as interesting as possible!

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[1] Charles Bennett, Michael Turner, and Martin White, "The Cosmic Rosetta Stone," *Physics Today*, Nov. 1997, pp. 32-38.

[2] J.L. Heilbron, *The Dilemmas of an Upright Man: Max Planck as Spokesman for German Science* (Univ. of Calif., 1986), pp. 14-21.

[3] For examples of the influence of Planck's work, seen from reprints or translations of primary sources, see John Stachel, *Einstein's Miraculous Year: Five Papers that Changed the Face of Physics* (Princeton Univ. Press, 1998; or Niels Bohr, *On the Constitution of Atoms and Molecules* (Munksgaard, Copenhagen, 1963).

[4] A. Penzias and R. Wilson, *Astrophysical Journal*, **142**, 419 (1965).

[5] For a simplified model of the early universe, see recent issues of *Radiations*: "Big Bang Cosmology, Part 1: The Expansion of the Universe" (Winter 2000, pp. 20-24), and "Big Bang Cosmology, Part 2: Nucleosynthesis and the Background Radiation" (Spring 2001, pp. 15-19).

[6] S. Weinberg, *Gravitation and Cosmology* (Wiley, 1972), p. 517.

[7] "COBE Satellite Finds No Excess in the Cosmic Microwave Spectrum," *Physics Today*, March 1990, pp. 17-20; "COBE Measures Anisotropy in Cosmic Microwave Background Radiation," *Physics Today*, June 1992, pp.17-20; "Cosmic Microwave Observations Yield More Evidence of Primoridal Inflation," *Physics Today*, July 2001, pp. 16-18..

[8] John A. Peacock, Cosmological Physics (Cambridge, 1999), p. 288.

[9] George Smoot and Keay Davidson, *Wrinkles in Time* (Morrow, 1993).

[10] Maxima web site: physics7.berkeley.edu/group/cmb/gen.html

[11] Boomerang web site: astro.caltech.edu/mc/boom/boom.html. See also F. Piacentini et. al., "The BOOMERANG North American Instrument: A Balloon-borne Bolometric Radiometer Optimized for Measurements of Cosmic Background Radiation Anisotropies from 0.3K to 4K," *Astrophysical Journal Supp*., **138**, pp. 315-356 (Feb. 2002).

[12] DASI web site: astro.uchicago.edu/dasi/ Other web sites are listed in Ref. 1.

[13] Michael Turner and Wayne Hu, NSF Chautauqua Course, "Quantum Fuzz and the Accelerating Universe," November 2001, Univ. of Chicago. See http://background.uchicago.edu/~whu/intermediate/prologue.html

[14] C. Pryke et.al., "Cosmological Parameter Extraction from the First Season of Observations with DASI," *Astrophysical Journal*, March 1, 2001, (http://arxiv.org/abs/astro-ph0104490). Another recent example of extracting cosmological information from the CBR is found in Luca Amendola, "The Dependence of Cosmological Parameters Estimated from the Microwave Background in Non-Gaussianity," *The Astrophysical Journal*, **569**, (April 20, 2002), pp. 595-599.

[15] J.L. Heilbron, ref. 2, p. 21.

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